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HEAT TRANSFER IN LAMINAR PLANE CHANNEL FLOW WITH UNIFORM SUCTION OR INJECTION

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Abstract—Consideration is given to heat transfer in a developed laminar incompressible flow with constant physical properties in a two-dimensional channel with porous walls having constant temperature. Several asymptotic solutions of the energy equation for small and large wall Peclet numbers and large Prandtl numbers are obtained. The computed results of the local Nusselt number distribution in both the thermal stabilization and channel entrance regions are generalized for all Prandtl numbers by a single relation in the form of the relative heat transfer law.

NOMENCLATURE

x, y,	axial and transverse coordinates;
u_x, u_y,	axial and transverse velocity components;
U_1,	mean velocity at the channel entrance;
h,	duct half-width;
T,	temperature;
T_1, T_w,	inlet and wall temperature, respectively;
v,	kinematic viscosity;
a,	thermal diffusivity;
Pr,	$= v/a$, Prandtl number;
V,	suction or injection velocity (positive for suction);
R,	$= Vh/v$, Reynolds number;
Pe_v,	$= 2Vh/\alpha$, suction or injection Peclet number;
P,	$= \|Pe_v\|$, suction or injection rate;
η,	$= y/h$, dimensionless transverse coordinate;
X,	$= xa/h^2U_1$, dimensionless axial coordinate;
z,	dimensionless axial coordinate;
θ,	$= (T-T_w)/(T_1-T_w)$, dimensionless temperature;
θ_b,	bulk temperature;
Nu,	Nusselt number;
F, f,	velocity functions;
δ,	distance from the axis where $f = 0$;
Φ_n, ϕ_n,	eigenfunctions;
μ_n,	eigenvalues;
A_n,	constants;
ω, g,	coefficients in equation (22);
ψ, b,	relative heat transfer coefficient and suction (injection) parameter, respectively;
C_0, C, B_0, K,	integration constants.

Subscripts

in,	injection;
s,	suction;
n, m, k, l,	number of the term in expansion.

1. INTRODUCTION

SINCE the publication of Berman's work [1], the problems of liquid flow in channels and pipes with porous walls have received much attention from investigators due to increasing use of suction and injection in modern technology.

Heat transfer in laminar flow through circular tube with injection was first discussed in [2] subject to the boundary conditions of the first kind. The energy equation was solved by the method of separation of the variables, with the eigenvalue problem being integrated by expanding the eigenfunctions into a power series along the radial coordinate. In [3, 4], the methods of the perturbation theory were employed to find the first eigenfunction of the respective Sturm–Liouville problem at low and high suction or injection rates. Calculations of stabilized heat transfer in a circular tube were carried out in [5] and extended to the thermal entrance region in [6]. Numerical solutions of the energy equation by the Fourier method for circular tubes and two-dimensional rectangular ducts at boundary conditions of the first and second kinds, as well as a number of asymptotic expansions of the solution for strong injection were obtained in [7]. Calculations of heat transfer for the thermal entrance region of a two-dimensional channel, with hydrodynamic flow stabilization and without, were obtained by a finite-difference method in [8, 9]. The influence of variable physical properties of the fluid on heat transfer in a two-dimensional flat channel is studied in [10]. Analytical solutions of the linearized motion and energy equations were given in [11] as a series of eigenfunctions expressed in terms of hypergeometrical functions.

In this work, asymptotic solutions of the energy equation have been obtained which make it possible from the results of numerical calculations to establish a correlation between the relative heat transfer coefficient and the suction (injection) parameter.

2. THE SOLUTION BY THE METHOD OF SEPARATION OF THE VARIABLES

The energy equation for a steady-state flow of incompressible fluid with constant physical properties in a plane–parallel channel without regard for axial heat conduction is

$$u_x \frac{\partial T}{\partial x} + u_y \frac{\partial T}{\partial y} = a \frac{\partial^2 T}{\partial y^2}. \quad (1)$$

In the case of axial symmetry, with the temperature at the inlet cross-section and channel walls being constant, the boundary conditions are

$$x=0 \quad T=T_1, \quad y=0 \ \frac{\partial T}{\partial y} = 0, \quad y=h \quad T=T_w. \ (2)$$

The axial and transverse velocities for hydrodynamically stabilized flow may be expressed in terms of a single function $F(\eta; R)$ [1]

$$u_x=(U_1 - V \cdot x/h)F'(\eta), \quad u_y = V \cdot F(\eta) \quad (3)$$

where F is found by solving the ordinary differential equation

$$F''' + R(FF'' - F'^2) = K$$
$$\eta=0 \quad F=F''=0, \quad \eta=1 \quad F'=0 \quad F=1. \quad (4)$$

At small values of the parameter R, the solution of equation (4) can be given as the following expansion [1]

$$F(\eta; R) = F_0(\eta) + RF_1(\eta) + O(R^2), \quad (5)$$

where

$$F_0 = \frac{3}{2}\eta - \frac{\eta^2}{2}$$

corresponds to plane Poiseuille flow.

Taking into account equation (3), the energy equation (1) can be written in terms of dimensionless variables as

$$\left(1 - \frac{Pe_v X}{2}\right) F'(\eta; R) \frac{\partial \theta}{\partial X} + \frac{Pe_v}{2} F(\eta; R) \frac{\partial \theta}{\partial \eta} = \frac{\partial^2 \theta}{\partial \eta^2}. \quad (6)$$

Let us transform equation (6) from the coordinate x to a new axial variable

$$z = -\frac{2}{Pe_v} \ln\left(1 - \frac{Pe_v X}{2}\right) \quad (7)$$

changing from 0 to ∞ for both injection and suction. The relationship between the variables z and X is shown in Fig. 1.

Then, equation (6) subject to the boundary conditions (2) will be written as

$$F'(\eta; R) \frac{\partial \theta}{\partial z} + \frac{Pe_v}{2} F(\eta; R) \frac{\partial \theta}{\partial \eta} = \frac{\partial^2 \theta}{\partial \eta^2}$$
$$z=0 \quad \theta=1, \eta=0 \quad \frac{\partial \theta}{\partial \eta} = 0, \eta = 1 \quad \theta=0. \quad (8)$$

The Nusselt number is determined from

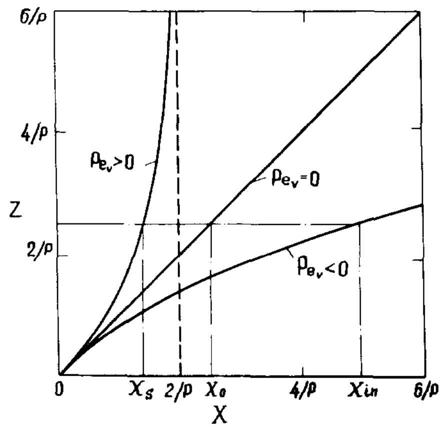

FIG. 1. The relationship between the variables z and X.

$$Nu = -\frac{2}{\theta_b}\left(\frac{\partial \theta}{\partial \eta}\right)_{\eta=1},$$

where

$$\theta_b = \int_0^1 \theta F' \, d\eta$$

is mean flow bulk temperature which changes along the channel length according to the one-dimensional energy conservation equation

$$\frac{d\theta_b}{dz} = \frac{(Pe_v - Nu)}{2} \theta_b. \quad (9)$$

The solution of the energy equation (8) can be presented as a series of eigenfunctions

$$\theta = \sum_{n=1}^{\infty} A_n \Phi_n(\eta) \exp(-\mu_n z) \quad (10)$$

where Φ_n and μ_n are found from the solution of the Sturm–Liouville problem

$$\frac{d^2 \Phi_n}{d\eta^2} - \frac{Pe_v}{2} F \frac{d\Phi_n}{d\eta} + \mu_n F' \Phi_n = 0,$$
$$\Phi_n'(0) = \Phi_n(1) = 0. \quad (11)$$

By inserting

$$\Phi_n = \exp\left(\frac{Pe_v}{4} \int_0^\eta F \, d\lambda\right) \phi_n \quad (12)$$

into equation (11), we obtain the following eigenvalue problem to determine ϕ_n and μ_n

$$\frac{d^2 \phi_n}{d\eta^2} + \left[\left(\mu_n + \frac{Pe_v}{4}\right) F' - \frac{Pe_v^2}{16} F^2\right] \phi_n = 0,$$
$$\phi_n'(0) = \phi_n(1) = 0. \quad (13)$$

The constants A_n in (10) are found from the expression

$$A_n = \int_0^1 \exp\left(\frac{Pe_v}{4} \int_0^\eta F \, d\lambda\right) F' \phi_n \, d\eta \Big/ \int_0^1 F' \phi_n^2 \, d\eta.$$

At high Prandtl numbers, $Pr \gg 1$, and the Peclet numbers for injection (suction) that meet the condition

$P/Pr \ll 1$, injection or suction has a minor effect on the flow hydrodynamics, since the parameter $R = Pe_v/2Pr$ in this case is small, it is possible to disregard all terms after the first one in equation (5), i.e. to assume that $F = F_0$. This allows one to establish a correlation between the solutions of the energy equation for suction and injection occurring at the same rate, that is at the same values of P. As seen from equation (13), the only difference between the suction and injection at the same values of P lies only in the sign in front of the P in $\mu_n + Pe_v/4$. It follows from equation (13) that at high Prandtl numbers, when F is independent of Pe_v, the eigenfunctions and eigenvalues for suction and injection of the same intensity are related as

$$\Phi_{n_{in}} = \exp\left(-\frac{P}{2}\int_0^\eta F_0\,\mathrm{d}\lambda\right)\Phi_{n_s}, \mu_{n_{in}} = \mu_{n_s} + \frac{P}{2}. \quad (14)$$

Taking into account equation (14) one can establish a correlation between the bulk temperatures and local Nusselt numbers for suction and injection that correspond to the same value of P

$$\theta_{b_{in}}(z;P) = 1 - \exp\left(-\frac{PZ}{2}\right)\left[1-\theta_{b_s}(z;P)\right] \quad (15)$$

$$Nu_{in}(z;P) = Nu_s(z;P) - P. \quad (16)$$

The relationship between θ_b and Nu in equations (15) and (16) is valid at equal values of z on the left- and the right-hand side in the equations which corresponds to different X_{in} and X_s (Fig. 1) related by

$$X_{in} = \frac{X_s}{1-PX_s/2}.$$

Consider the solution of the energy equation at low velocities of suction and injection when the term of the order of Pe_v^2 in equation (13) can be neglected. Then equation (12) is solved as

$$\theta = \exp\left(\frac{Pe_v}{4}\int_0^\eta F\,\mathrm{d}\lambda\right)\sum_{n=1}^{\infty}\frac{A_{n0}\phi_{n0}(\eta)}{1-Pe_v/4\mu_{n0}} \times \exp\left[-\left(\mu_{n0}-\frac{Pe_v}{4}\right)z\right] \quad (17)$$

where

$$A_{n0} = \int_0^1 F'\phi_{n0}\,\mathrm{d}\eta \Big/ \int_0^1 F'\phi_{n0}^2\,\mathrm{d}\eta$$

and ϕ_{n0} and μ_{n0} are eigenfunctions and eigenvalues of the following Sturm–Liouville problem

$$\frac{\mathrm{d}^2\phi_{n0}}{\mathrm{d}\eta^2} + \mu_{n0}F'\phi_{n0} = 0, \quad \phi'_{n0}(0) = \phi_{n0}(1) = 0. \quad (18)$$

The eigenvalues μ_n in the first linear approximation on Pe_v are related to μ_{n0} through

$$\mu_n = \mu_{n0} - \frac{Pe_v}{4}.$$

From equation (17) one can determine the Nusselt number at low values of Pe_v

$$Nu(z;Pe_v,R) = Nu_0(z;R) + \frac{Pe_v}{2} \quad (19)$$

where Nu_0 is found by solving the energy equation without regard for the transverse convection

$$F'(\eta;R)\frac{\partial\theta_0}{\partial z} = \frac{\partial^2\theta_0}{\partial\eta^2}. \quad (20)$$

The coupling between X and X_0 (Fig. 1), to which Nu and Nu_0 in (19) should be related, is determined by

$$X = \frac{2}{Pe_v}\left[1-\exp\left(-\frac{Pe_vX_0}{2}\right)\right]. \quad (21)$$

Thus, equations (17) and (19) allow one to express the temperature distribution and Nusselt number at a small value of Pe_v in terms of the characteristics of a more simple problem (18) or (20). For the Poiseuille velocity profile the values of ϕ_{n0}, μ_{n0}, A_{n0} and Nu are found from the solution of the Nusselt–Graetz plane problem [12].

Let us now pass to the solution of the problem for strong suction or injection. We shall derive the expressions for the first eigenfunctions and eigenvalues in extreme cases of $Pe_v \to \pm\infty$. Equation (11) will then contain a small parameter at the higher derivative $1/Pe_v$. Since at $P \to \infty$ the order of the differential equation decreases, there is no need for the extreme eigenfunctions to satisfy the two boundary conditions in (11). In order to determine the limiting eigenfunctions Φ_1^* and eigenvalues μ_1^*, these can be given as a series in a small parameter

$$\Phi_1^* = \sum_{k=0}^{\infty}\left(\frac{Pe_v}{2}\right)^{-k}\omega_k, \quad \mu_1^* = \sum_{k=0}^{\infty}\left(\frac{Pe_v}{2}\right)^{1-k}g_k. \quad (22)$$

Substituting (22) into (11) and equating the terms with the like powers of Pe_v we obtain a chain of equations to find ω_k and g_k

$$F\frac{\mathrm{d}\omega_0}{\mathrm{d}\eta} - g_0F'\omega_0 = 0 \quad (23)$$

$$F\frac{\mathrm{d}\omega_k}{\mathrm{d}\eta} - g_0F'\omega_k = \frac{\mathrm{d}^2\omega_{k-1}}{\mathrm{d}\eta^2} + g_kF'\omega_0 + F'\sum_{l,n=0}^{l+n=k} g_l\omega_n, \quad k \geq 1. \quad (24)$$

For suction, the solution of equation (23) is

$$\omega_{0_s} = C_0, \; g_{0_s} = 0.$$

Then, from (24) we find

$$\omega_{1_s} = g_1 \ln F + C_1,$$

and since $F(0) = 0$, it follows that $g_{1s} = 0$. Similarly, it can be shown that $g_{k_s} = 0$ at $k \geq 2$.

In the case of injection, equation (23) is satisfied, with due regard for the rules of differentiation of generalized functions [13], by the δ-function

$$\omega_{0_{in}} = B_0\delta(\eta), \quad g_{0_{in}} = -1.$$

From (24) we obtain

$$g_{1_{in}}F'(0) = \int_0^1 F\omega'_{1_{in}}\,d\eta + \int_0^1 F'\omega_{1_{in}}\,d\eta = F\omega_{1_{in}}\Big|_0^1 = 0.$$

Hence $g_{1_{in}} = 0$, since $F'(0) \neq 0$; in a similar fashion we can find that $g_{k_{in}} = 0$ at $k \geqslant 2$.

Thus, all the first eigenfunctions normalized to

$$\int_0^1 \Phi_1\,d\eta = 1$$

are located between two extreme profiles; $\Phi_1 = 1$ at $Pe_v \to \infty$ and $\Phi_1 = \delta(\eta)$ at $Pe_v \to -\infty$. The eigenvalues μ_1 are determined by the asymptotic expressions

$$\mu_1 = o(Pe_v^{-n}) \text{ at } Pe_v \to \infty,\ \mu_1 = -\frac{Pe_v}{2} + o(Pe_v^{-n}) \text{ at } Pe_v \to -\infty \qquad (25)$$

where $n = 0, 1, 2, \ldots$.

It follows from (25) that with the growth of P, μ_1 very rapidly approaches its extreme values equal to zero for suction and $-Pe_v/2$ for injection. It can be shown that the law governing this approach has an exponential character.

In order to derive asymptotic expressions for eigenfunctions and eigenvalues at any n in the case of strong suction or injection, we shall use the WKB method [14]. At large P the function in the square brackets in equation (13)

$$f = \left(\mu_n + \frac{Pe_v}{4}\right)F' - \frac{Pe_v^2}{16}F^2$$

vanishes at a small distance from the axis

$$\delta = \frac{4}{P}\sqrt{\left[\frac{\mu_n + Pe_v/4}{F'(0)}\right]}. \qquad (26)$$

Assuming that within the region $\delta < \eta < 1$, where $f < 0$, an exponentially increasing term is lacking in the solution of equation (13), we set

$$\phi_n = \frac{K}{\sqrt[4]{-f}}\exp\left[-\int_0^\eta \sqrt{(-f)}\,d\lambda\right]. \qquad (27)$$

Equation (27) conforms asymptotically to the boundary condition $\phi_n(1) = 0$ with an accuracy of $\exp(-P)/P^{1/2}$. In the range $0 < \eta < \delta$, the solution of equation (13), matched with (27), has the form

$$\phi_n = \frac{2K}{\sqrt[4]{f}}\cos\left[\int_\eta^\delta \sqrt{(f)}\,d\lambda - \frac{\pi}{4}\right]. \qquad (28)$$

The solution (28) should satisfy the boundary condition $\phi'_n(0)=0$. Hence, accounting for $f'(0)=0$ we obtain

$$\sin\left[\int_0^\delta \sqrt{(f)}\,d\lambda - \frac{\pi}{4}\right] = 0$$

or for (26)

$$\sin\left[\frac{\pi}{P}\left(\mu_n + \frac{Pe_v}{4}\right) - \frac{\pi}{4}\right] = 0. \qquad (29)$$

From equation (29), we find the limiting expression for the eigenvalues

$$\lim_{P\to\infty} \mu_n = \frac{P - Pe_v}{4} + (n-1)P. \qquad (30)$$

As follows from (30), μ_n is independent of the velocity distribution (F function) at $P \to \infty$ and depends only on the suction or injection rate and the ordinal number n.

The results of numerical solution of equation (13) obtained for the Poiseuille velocity profile confirm the asymptotic expression (30) shown by dashed lines in Fig. 2. Figure 3 depicts the first three eigenfunctions of equation (13); it is seen that at high values of P the region of oscillating ϕ_n shifts toward the axis, while in the wall region $\phi_n \to 0$. The nature of the obtained distribution of eigenfunctions agrees well with the asymptotic solutions (27) and (28).

Equations (25) and (30) yield the relationships for Nu under intense suction or injection

$$\begin{aligned} Nu &= Pe_v + o(Pe_v^{-n}) \quad \text{at } Pe_v \to \infty, \\ Nu &= o(Pe_v^{-n}) \quad \text{at } Pe_v \to -\infty, \end{aligned} \qquad (31)$$

where $n = 0, 1, 2, \ldots$.

Equations (31) specify the limiting functions for Nu derived in [4].

3. CALCULATION BY THE FINITE-DIFFERENCE METHOD

In order to obtain a numerical solution of equation (8), an implicit two-layer, six-point difference scheme was employed with the use of the factorization method for each layer. The function F was determined by integrating equation (4).

Figure 4 shows the calculated local Nusselt number at different values of the parameters Pe_v and Pr. With

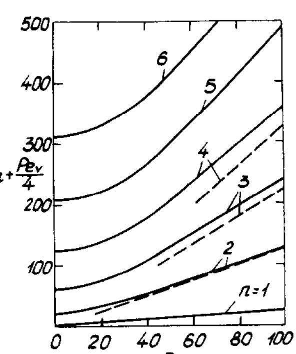

FIG. 2. Dependence of eigenvalues on the parameter P.

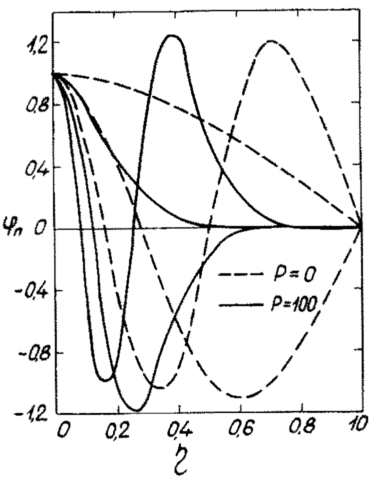

FIG. 3. Distribution of eigenfunctions.

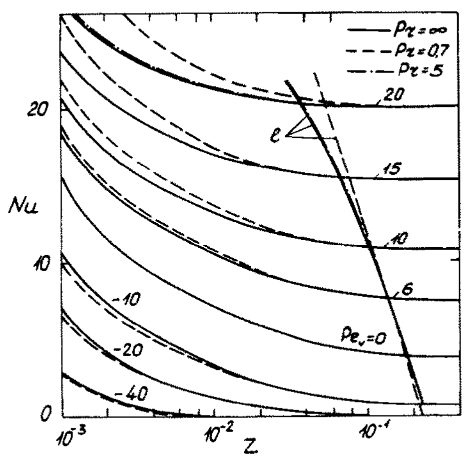

FIG. 4. Variation of Nu number along the channel.

an increasing injection rate the Nu number tends asymptotically toward zero for all z, while with an increasing suction rate it approaches Pe_v which agrees with (31). The curves l in Fig. 4 show the boundaries of the thermally stabilized region found from the condition $Nu = 1.01\, Nu_\infty$.

The larger effect of the decreasing Pr number on Nu is attributed to a rise in the parameter $R = Pe_v/2Pr$ at a fixed Pe_v. In the case of suction the fullness of the axial velocity profile increases with R [15], on account of which for lower values of Pr there are higher values of Nu. On the other hand, in the case of injection the fullness of the profile somewhat decreases as compared to the Poiseuille one, though not much, which is responsible for the reversed effect of Pr on Nu. As seen from Fig. 4, Nu_∞ is actually independent of Pr (or R) in the thermally stabilized region at $Pr \geqslant 0.7$ which was also pointed out in [7]. In the thermal entrance region at low Prandtl numbers this effect can be substantial for suction, but as soon as $Pr \geqslant 5$ it becomes insignificant.

Figure 5 shows variation in the fluid bulk temperature along the channel. In the case of injection, θ_b drops more abruptly than it does in a channel with impermeable walls which is accounted for by some additional loss of heat by the main flow for heating the cold injected liquid (if the temperature at the channel inlet exceeds that of the wall). Conversely, in the case of suction the liquid, which leaves the channel at the wall temperature, gives up, according to (9), the excess of heat $Pe_v\theta_b/2$ to the main flow and θ_b varies more slowly than in a channel with impermeable walls. According to (9), at a high suction rate, $Nu \rightarrow Pe_v$, the bulk temperature changes hardly at all, and over the whole channel, except for a narrow wall region, there is virtually an isothermal flow with a temperature equal to that at the channel inlet.

4. THE RELATIVE HEAT TRANSFER COEFFICIENT AS A FUNCTION OF THE SUCTION (INJECTION) PARAMETER

In the theory of a turbulent boundary layer wide usage has been gained by the relative law of heat and mass transfer $\psi = f(b)$ [16], where $\psi = Nu/Nu_0$ is the relative heat transfer coefficient, $b = Pe_v/Nu_0$ is the suction or injection parameter and Nu_0 is the Nusselt number for the flow past an impermeable plate. The relative variables ψ and b allow the experimental and computed data on heat transfer to be correlated by a virtually universal relation which depends only weakly on Re and Pr numbers, the effect of which on Nu is taken into account by the normalizing coefficient $Nu_0(Re, Pr)$. Similarly, with a suitable choice of Nu, one can correlate the predicted results on heat transfer for laminar flows in channels.

In order to justify the selection of Nu_0 in the relative heat transfer law, let us consider the various factors responsible for the effect of suction and injection on heat transfer. As follows from equation (6), there are three reasons for the effect of suction (injection) on heat transfer: first, the change in the flow rate with

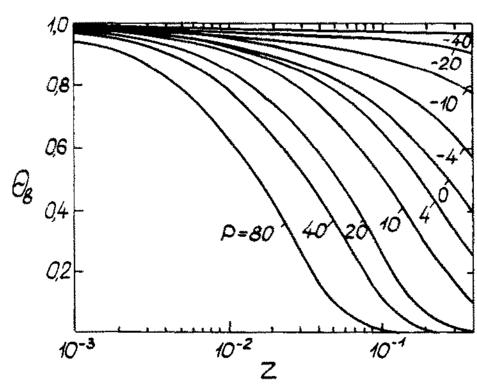

FIG. 5. Variation of the mean bulk temperature along the channel at $Pr = 0.7$.

distance along the channel which is taken into account by the factor $(1-Pe_v X/2)$ in the first term; second, the suction (injection)-induced distortion of the axial velocity profile, and third, lateral convective heat transfer [the second term of equation (6)].

The effect of the first factor may be obviated by transforming to a new axial coordinate, z, according to (7). Then, the first term in equation (8) which stands for the axial heat convection will have the same form as that for a channel with a constant flow rate. Therefore, comparison between the numbers Nu and Nu_0 should be made in different cross-sections, X and X_0, which correspond to one and the same value of z and which are interrelated by equation (21).

The dependence of heat transfer on a change in the axial velocity profile characterized by the parameter R (or Pr at a fixed Pe_v) can be determined separately by solving equation (20). The results of computation of $Nu_0(z)$ at various R are given in Fig. 6. The extreme cases when $|R| = \infty$ correspond to a slug velocity profile, $F' = 1$, for suction and to a cosine profile, $F' = \pi/2 \cos(\pi\eta/2)$, for injection [15].

If now for Nu_0 we take $Nu_0(z, R)$ from the solution of equation (20), the relative heat transfer coefficient $\psi = Nu(z, Pe_v, R)/Nu_0(z, R)$ will then describe a direct influence of suction or injection on heat transfer caused by the lateral convection alone. Let the relative suction (injection) parameter be $b = Pe_v/Nu_0(z, R)$. Then the asymptotic correlations (19) and (31) will take the form

$$\psi(b\to 0) = 1 + \frac{b}{2}, \quad \psi(b\to\infty) = b + o(b^{-n}),$$
$$\psi(b\to -\infty) = o(b^{n}). \quad (32)$$

The relative variables ψ and b have been introduced with the object of deriving a single relationship which could correlate the predicted results on heat transfer for all Pr. The effect of Pr (or R) on Nu is accounted for by the coefficient $Nu_0(R)$. As seen from (32), the function $\psi(b)$ is really independent of Pr at low and high values of b. Since $\psi(b)$ does not depend on Pr within the whole range of the parameter b, then, by equation (16), the following condition should be satisfied

$$\psi_{in}(|b|) = \psi_s(|b|) - |b|. \quad (33)$$

The results of numerical calculation of the heat transfer modes (Fig. 7), for both the stabilized region and inlet section, over the whole range of Prandtl numbers are rather well correlated by the Mickly–Spalding equation

$$\psi = \frac{b \exp b}{\exp b - 1} \quad (34)$$

which satisfies equations (32) and (33).

It should be noted that in determining ψ and b for a stabilized heat transfer at $Pr \geqslant 0.7$ one can replace $Nu_{\infty 0}(R)$ by $Nu_{\infty 0}(R = 0) = 3.77$ [12], i.e. as already remarked, the hydrodynamic effect of suction and injection on heat transfer can be neglected as compared with the thermal effect caused by convection and characterized by the parameter Pe_v. Figure 8 compares the predictions of equation (34) at $Nu_{\infty 0} = 3.77$ (solid line) with the numerical solution at $Pr = 0.7$ (dotted line); the crosses show the results taken from [8] for $Pr = 0.72$. The both curves virtually coincide for suction and agree sufficiently well for injection. With an increasing Pr, this agreement becomes even better. As seen from Fig. 6, in the case of injection the difference between $Nu_0(z)$ in the inlet section is small even for $R = 0$ and $R = -\infty$. Therefore, in the case of injection

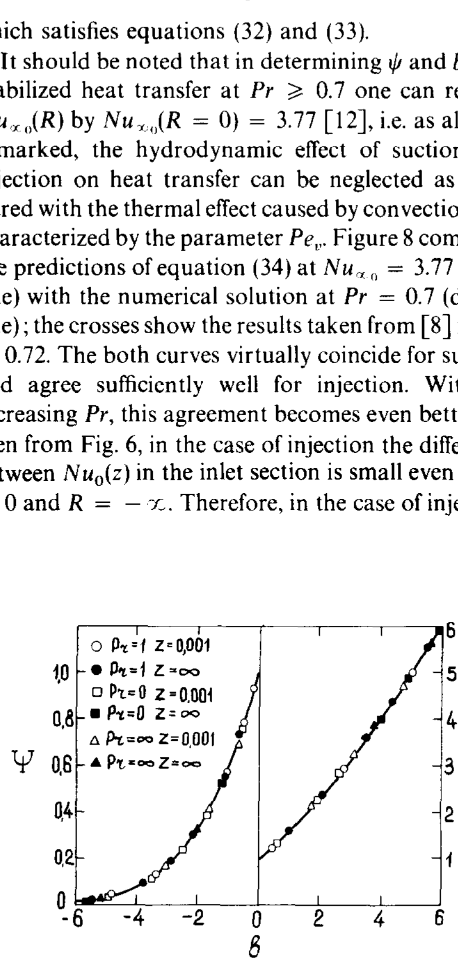

FIG. 7. Dependence of the relative heat transfer coefficient on the parameter b.

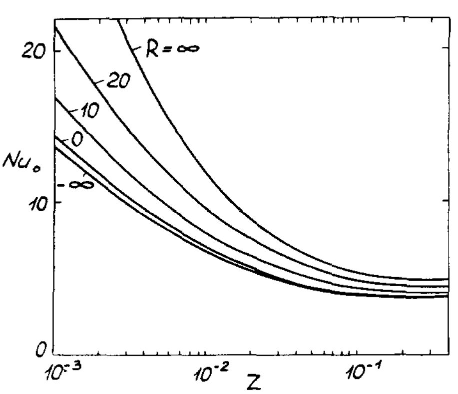

FIG. 6. Variation of Nu_0 along the channel.

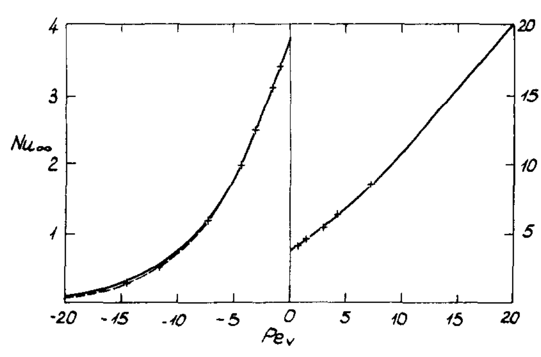

FIG. 8. Dependence of Nu on Pe_v over the thermally stabilized region.

one can assume $Nu_0(z, R) = Nu_0(z, R = 0)$ for all the Prandtl numbers throughout the entire flow. For suction, this difference can be rather appreciable in the thermal entrance section and should be taken into account at small Prandtl numbers.

We conclude by noting that the results obtained for a plane channel can be readily extended to a circular tube flow except for the range of suction velocities within which there is no hydrodynamically developed flow [18].

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TRANSFERT THERMIQUE DANS UN ECOULEMENT LAMINAIRE EN CONDUITE AVEC SUCCION OU INJECTION UNIFORME

Résumé—On étudie le transfert thermique dans un écoulement laminaire et incompressible à propriétés constantes dans un canal bidimensionnel à parois poreuses isothermes. On obtient des solutions asymptotiques de l'équation de l'énergie pour des petits et des grands nombres de Péclet et des grands nombres de Prandtl. Les résultats calculés du nombre de Nusselt local dans la région thermiquement stabilisée et à l'entrée sont généralisés pour tous les nombres de Prandtl par une relation unique.

WÄRMEÜBERGANG BEI LAMINARER STRÖMUNG IN EINEM EBENEN KANAL BEI GLEICHFÖRMIGER ABSAUGUNG ODER EINSPRITZUNG

Zusammenfassung—Betrachtet wird der Wärmeübergang bei ausgebildeter laminarer inkompressibler Strömung mit konstanten Stoffwerten in einem zweidimensionalen Kanal mit porösen Wänden, die konstante Temperatur haben. Einige Näherungslösungen der Energiegleichung für kleine und große Peclet-Zahlen im Wandbereich sowie große Prandtl-Zahlen werden erhalten. Die berechneten Ergebnisse der örtlichen Nusselt-Zahl-Verteilung sowohl in den Gebieten der thermischen Stabilisierung als auch im Kanaleintrittsbereich werden für alle Prandtl-Zahlen durch eine einzige Beziehung in Form des relativen Wärmeübergangsgesetzes allgemeingültig wiedergegeben.

ТЕПЛООБМЕН ПРИ ЛАМИНАРНОМ ТЕЧЕНИИ В ПЛОСКОМ КАНАЛЕ С РАВНОМЕРНЫМ ОТСОСОМ ИЛИ ВДУВОМ ЖИДКОСТИ

Аннотация — Рассматривается теплоперенос при ламинарном развитом течении несжимаемой жидкости с постоянными физическими свойствами в плоском канале с проницаемыми стенками, имеющими постоянную температуру. Получены асимптотические решения уравнения энергии при малых и больших числах Пекле для потока через стенку и при больших числах Прандтля. Результаты численных расчетов распределения локального числа Нуссельта как в области тепловой стабилизации, так и во входном участке для всех чисел Прандтля обобщаются единой зависимостью в форме относительного закона теплообмена.